NONLINEAR ANALYSIS OF SHORT FIBER COMPOSITE STRUCTURES

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ABSTRACT

Injection molded plastic parts filled with short fibers are used widely in industrial applications due to their enhanced stiffness-to-weight and strength-to-weight ratios , strength and low weight when compared to metals. These composite materials also provide many advantages in structural performance over their unfilled plastic counterparts. However, a difficulty arises when an engineer is looking to understand the structural performance benefits because it is linked to the fiber orientation which is linked to the injection molding processing conditions. Adding difficulty to the problem is that the material often behaves in a nonlinear fashion prior to failure. Although this process sounds quite cumbersome and tedious, an ongoing project at Autodesk has made great strides simplifying the process and creating a workflow where Injection Molding Process Analysts and Structural Analysts can collaborate to achieve a solution. The details of the mapping fiber orientation results from the process analysis to the structural analysis will be presented along with nonlinear structural analysis and comparisons of analytical results to new experimental data generated at Autodesk.

Injection Molding simulation including prediction of the orientation of fiber fillers has been widely used for over three decades to predict and troubleshoot mold filling and the final shape of injection molded parts including the effects of anisotropic shrinkage leading to part warp. Recently, with the increasing focus of automotive light-weighting to achieve carbon emission reductions, there has been an increasing need for design validation tools which go beyond the injection molding process and consider the structural performance of the molded part under load. For fiber filled plastics, the orientation of the fibers has a critical effect on the structural performance, so simulation tools are required which allow structural analysis to consider the effect of fiber orientation and the non-linear anisotropic response of the plastic. This paper demonstrates a new simulation tool which facilitates the easy integration of molding process data from a mold filling analysis into the non-linear anisotropic material model used during structural analysis. Comparison of the predicted and measured non-linear mechanical response is provided as well as a demonstration of the importance of the non-linear anisotropic material description for accurate analysis of a real part.

1. INTRODUCTION

The use of short fiber reinforcing fillers has become common place in an effort to achieve higher stiffness-to-weight and higher strength-to-weight ratios for injection molded plastic parts. Modern software tools such as Autodesk Simulation Moldflow Insight allow designers to efficiently and accurately predict the orientation of the reinforcing fibers throughout the molded part, in addition to predicting the warped shape of the room temperature part after ejection from the mold. With these types of software tools, designers are able to iteratively develop manufacturing processes that produce a fiber filled plastic part that satisfies design requirements for room temperature shape and initial elastic stiffness. However, to produce optimal designs for injection molded parts, the designer must also consider the in-service thermo-mechanical performance characteristics of the part.

For injection molded plastic parts that contain short fiber reinforcing fillers, prediction of the mechanical response is complicated by the fact that the elastic, plastic, and rupture responses of the composite material are highly anisotropic due to the local orientation of the reinforcing fibers[1], and these local fiber directions can vary throughout the injection molded part due to spatial variation of flow conditions during the injection molding process.[2] Thus an accurate simulation of the mechanical response of a fiber-filled, injection molded part requires a model that can *1)* accurately represent the anisotropic elastic, plastic and rupture response of the composite material as influenced by the local fiber direction, and 2) accurately account for the variation of local fiber direction throughout the part.[1]

To address these challenges, Autodesk is currently developing software that provides a seamless transition from the injection molding simulation to the nonlinear structural response simulation. Specifically, the software provides a seamless link between Autodesk Simulation Moldflow Insight (ASMI) and Autodesk Simulation Composites Analysis (ASCA). The key features of this software include:

1) Automated mapping of the ASMI-predicted fiber orientation distribution onto the finite element mesh that will be used for the ASCA nonlinear structural response simulation,
2) Enhancement of ASCA with a multiscale, progressive failure, constitutive model for short fiber filled plastic materials that accounts for plasticity and rupture of the matrix constituent material, resulting in a composite material that exhibits an anisotropic, nonlinear response, and
3) A robust material characterization process that uses relatively simple, measured experimental data of the short fiber filled plastic material to fit the parameters of the multiscale, progressive failure, constitutive model.

The remainder of the paper will provide a detailed discussion of these key software features, in addition to demonstrating the utility of the software in simulating the progressive failure response of fiber filled injection molded parts.

2. DEFINING THE FIBER ORIENTATION DISTRIBUTION

Autodesk Moldflow Simulation Insight (ASMI) is routinely used to predict the spatial distribution of fiber orientation in short fiber filled, injection molded plastic parts. The 2nd order fiber orientation tensor at a point essentially provides a statistical description (in the continuum sense) of the orientation of fibers that lie in the immediate neighborhood of the point in question.[3] The eigenvectors of the fiber orientation tensor provide the principal material directions of the fiber filled plastic which is idealized as an orthotropic material. The eigenvalues of the fiber orientation tensor provide a measure of the degree of orthotropy of the fiber filled plastic (i.e., the degree of randomness or alignedness of the reinforcing fibers). For example, a completely 3-D random fiber orientation would yield fiber orientation tensor eigenvalues of (1/3, 1/3, 1/3), while a perfectly aligned fiber orientation tensor would yield eigenvalues of (1, 0, 0).

The fiber orientation tensor can be used to operate on the constitutive matrix of a comparable composite material that contains perfectly aligned fibers to compute the anisotropic stiffness matrix of the actual composite material with the specified fiber orientation distribution (a process referred to as fiber orientation averaging.)[1] Gustev et al.[4] demonstrated that anisotropic stiffness matrices that are predicted with fiber orientation averaging compare very well with the anisotropic stiffness matrices that are obtained from finite element models that contain actual fiber distributions that correspond to the specified fiber orientation tensor. Since the fiber orientation averaging process is quite reliable, one can develop complex nonlinear material models (e.g., plasticity, damage, rupture) for a single, much simpler, perfectly-aligned material, and then use the fiber orientation averaging process to compute the tangent constitutive matrix for the real composite material with the specified fiber orientation distribution.

In a simulation of the mechanical loading of a short fiber filled, injection molded plastic part, knowledge of the orientation of the reinforcing fibers at each point in the model is critical for accurately predicting the mechanical response of the material at the point. Generally speaking, the finite element mesh that will be used to simulate the mechanical response of an injection molded part will be different from the finite element mesh that is used to simulate the injection molding process. Thus it is necessary to interpolate the injection molding simulation's predicted fiber orientation tensor distribution onto the mesh that will be used to simulate the mechanical response. The present code automatically performs this interpolation and shows the user a side-by-side comparison of fiber orientation distributed on the Moldflow mesh and the ASCA mesh. Figure 1 shows a closeup view of the finite element meshes that are used for the injection molding simulation (via Moldflow) and the subsequent structural loading simulation (via ASCA). Figure 2 shows a comparison of the distribution of average fiber direction predicted on the Moldflow mesh and automatically interpolated onto the ASCA mesh. Compared to the ASCA mesh, the Moldfow mesh used a much higher level of mesh refinement through the thickness of the part, hence the Moldflow mesh is shown with a much denser collection of fiber direction arrows than the ASCA mesh; however, it is clearly evident that both meshes are showing the same distribution of fiber orientation.

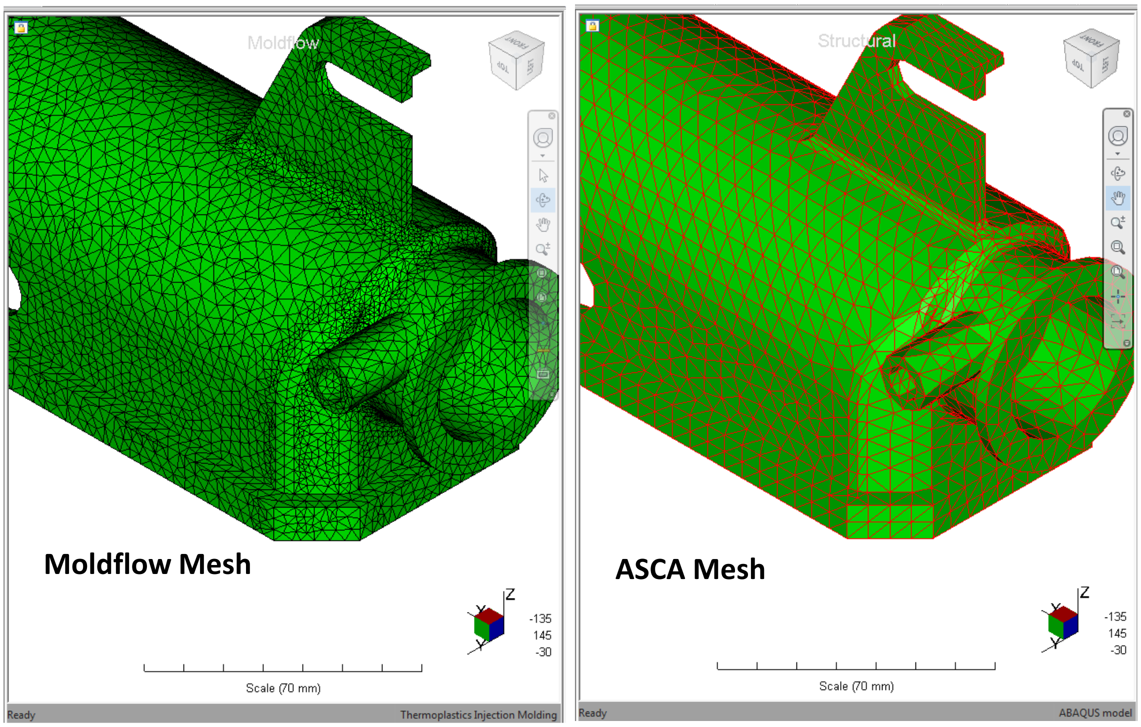

Figure 1. Comparison of the mesh used for simulation of the injection molding process (Moldflow) and the mesh used for structural loading simulation (ASCA)

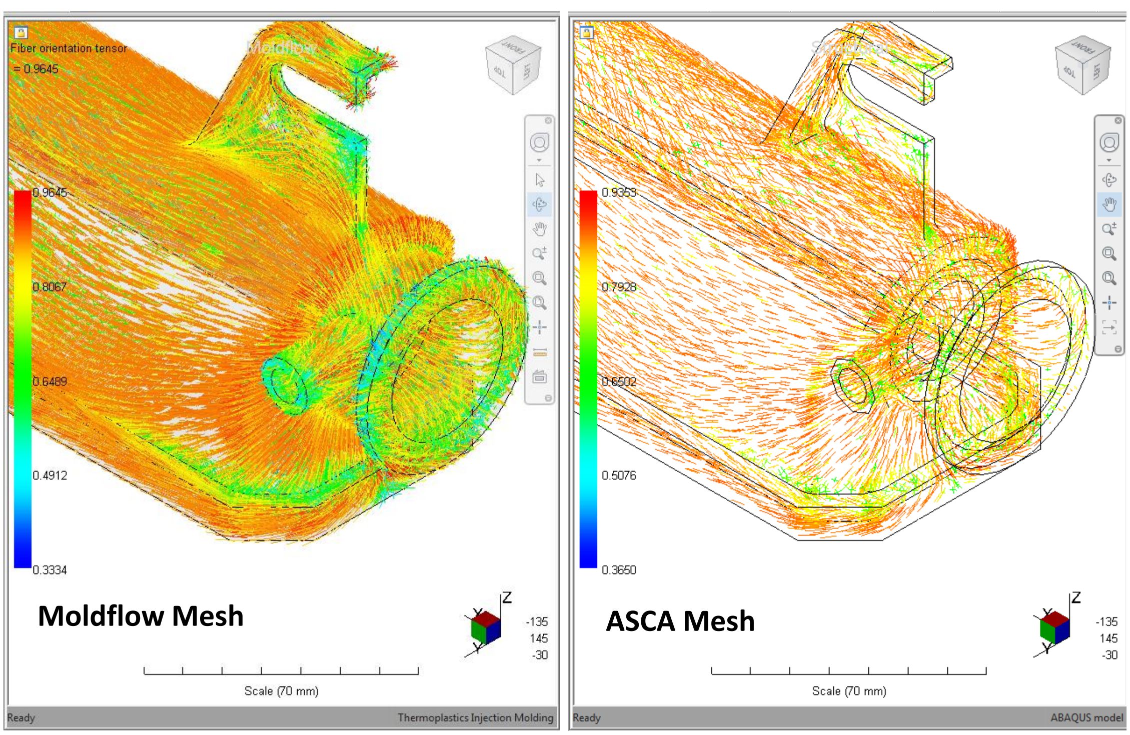

Figure 2. Comparison of the distribution of average fiber direction predicted on the Moldflow mesh and interpolated onto the ASCA mesh.

3. MULTISCALE PLASTICITY AND RUPTURE OF THE SHORT FIBER FILLED PLASTIC MATERIAL

Under mechanical loading, injection molded plastic parts typically exhibit a significant amount of plasticity prior to final rupture. The same is true for short fiber filled, injection molded plastic parts; however, the degree of plasticity exhibited by the material becomes strongly directionally dependent as the degree of fiber alignment increases from a random fiber orientation[5]. For short fiber filled, injection molded plastic parts that have a high degree of fiber alignment, the degree of plasticity exhibited prior to final rupture will depend strongly upon the direction of the loading relative to the average direction of the reinforcing fibers. In addition, as the degree of fiber alignment increases, the ultimate strength of fiber filled, injection molded plastic material will depend strongly upon the direction of the loading relative to the average direction of the reinforcing fibers. Furthermore, since the reinforcing fibers are short, the filled plastic material is able to rupture without actually breaking any of the reinforcing fibers; i.e., rupture occurs primarily by tearing of the plastic matrix material with some degree of short fiber pull-out.[6,7]

Based on the preceding description of the response characteristics of the short fiber filled plastic material, a multiscale material model was developed using the following assumptions and constraints:

- The short reinforcing fibers do not exhibit any plasticity or rupture, rather the fibers exhibit a simple linear elastic response,
- The plastic matrix constituent exhibits both plasticity and rupture,
- The idealized model's matrix plasticity and matrix rupture are intended to also account for any fiber/matrix debonding that occurs in the real material,
- All nonlinearity exhibited by the composite material is due to nonlinearity (plasticity and rupture) in the plastic matrix material,
- Plasticity and rupture of the plastic matrix constituent are driven by stress in the plastic matrix constituent as opposed to being driven by the homogenized stress in the composite material.
- The plasticity and rupture responses of the plastic matrix constituent are strongly dependent on the degree of alignedness of the reinforcing fibers.
- As the degree of fiber alignedness increases, the plasticity and rupture responses of the plastic matrix constituent become strongly dependent on the direction of loading relative to the average direction of the reinforcing fibers.

During an ASCA structural-level finite element simulation of mechanical loading of the short fiber filled plastic part, the predicted deformation of the part is based on the stiffness of the homogenized composite material. However, in order to predict plasticity and rupture of the matrix material, ASCA must decompose the finite element code's homogenized composite strain into the average strain in the matrix constituent material. Thus the multiscale material model must be capable of homogenizing the response of the evolving heterogeneous microstructure into composite-level stress, strain and stiffness, in addition to decomposing the composite-level strain into the average strain state in the plastic matrix constituent material. The structure of this multiscale material model is shown schematically in Figures 3 and 4 which illustrate the processes of homogenization and decomposition respectively. In Figure 3, the individual constituent properties are input into an incremental Mori-Tanaka micromechanical model[8-10] that is able to accommodate evolving matrix properties. The incremental Mori-Tanaka micromechanical model produces homogenized composite properties for the idealized, perfectly aligned material. These properties, in turn, are operated upon by the fiber orientation tensor to produce the homogenized composite properties for the real material with the actual fiber orientation distribution.

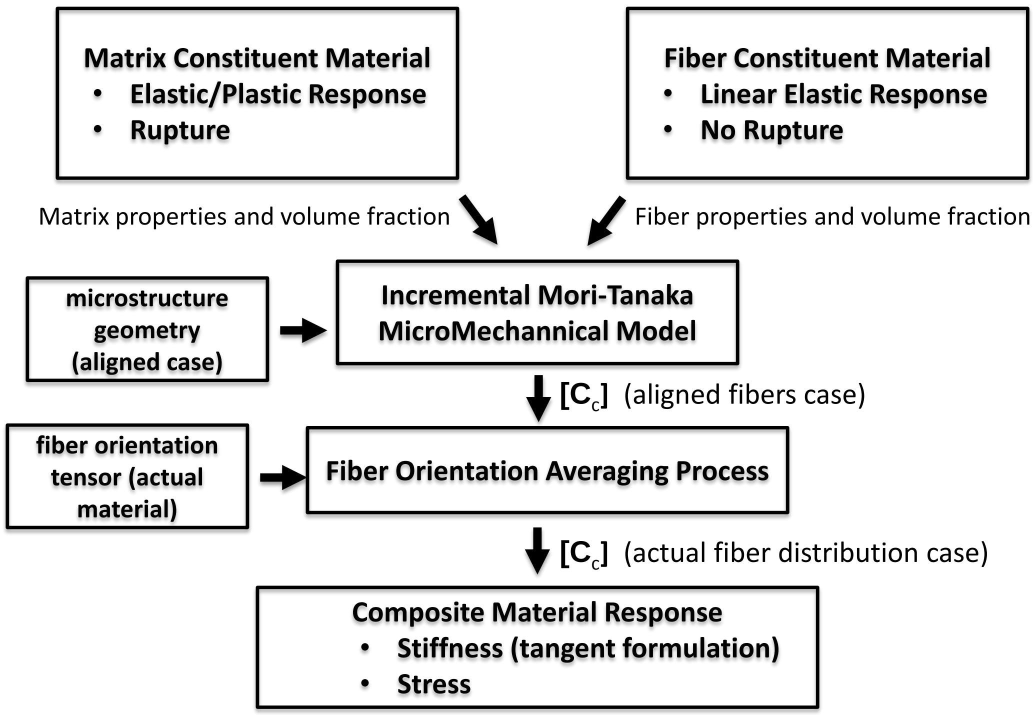

Figure 3. Schematic diagram of the multiscale material model's homogenization process.

Figure 4 provides a schematic diagram of the decomposition process that maps homogenized composite strain increments into the average strain increments in the plastic matrix constituent material. The decomposition process is taken from Nguyen et al.[11,12] and makes use of the instantaneous (tangent) constituent properties, the incremental Mori-Tanaka micromechanical model and the fiber orientation tensor. The computed average strain increment in the matrix constituent is used to drive the matrix plasticity model and predict the evolution of the matrix tangent modulus.

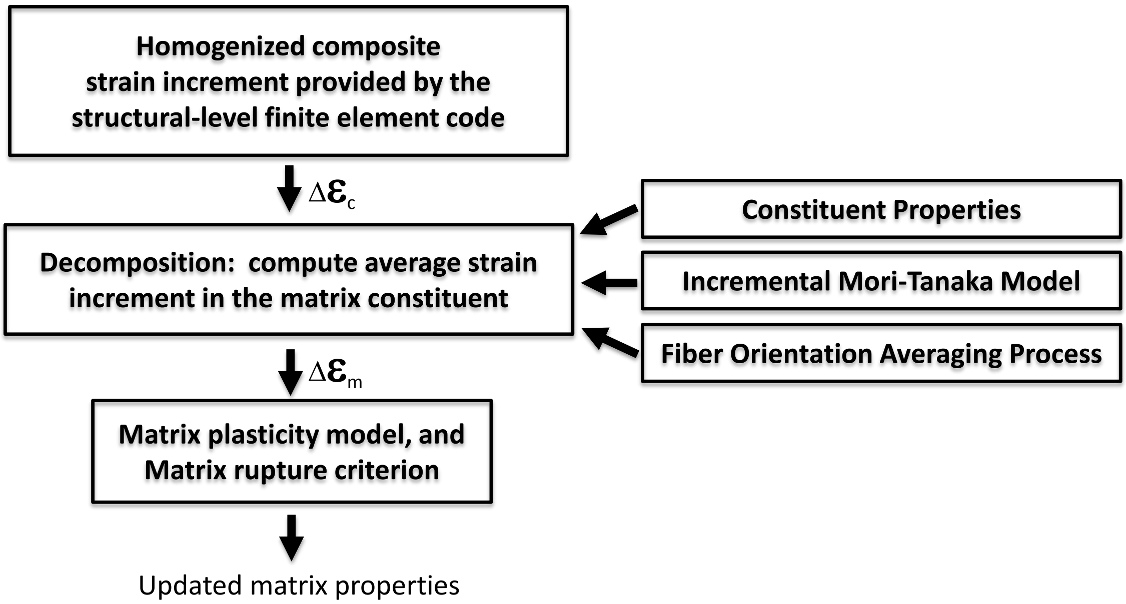

Figure 4. Schematic diagram of the decomposition process to convert the composite-level strain state into the average strain state in the matrix constituent which is used to update the matrix properties via the matrix plasticity model.

The response of the matrix constituent material is provided by a Ramberg-Osgood plasticity model[13] that has been enhanced to allow the predicted plastic response to exhibit sensitivity to the direction of the loading relative to the fiber direction. The effective hardened yield strength of the matrix constituent material can be expressed as

$$\sigma_Y^h(\varepsilon_{eff}^p) = E^{1/n}(\sigma_0)^{(n-1)/n}(\varepsilon_{p,eff})^{1/n} \tag{1}$$

where σ_0 and n are the typical material parameters that are used by the standard (isotropic) Ramberg-Osgood plasticity model, and $\varepsilon_{p,eff}$ is the effective plastic strain in the matrix constituent material. The yield function is satisfied when the effective stress in the matrix constituent matches the hardened yield strength.

$$f\left(\varepsilon_{p,eff}\right) \;=\; \sigma_{eff}\left(\varepsilon_{p,eff}\right) \;-\; \sigma_Y^h\left(\varepsilon_{p,eff}\right) \;= 0 \tag{2}$$

For isotropic materials, the effective (scalar) stress is often represented by the von Mises stress

$$\sigma_{eff} \equiv \sqrt{\frac{(\sigma_{11}-\sigma_{22})^2+(\sigma_{22}-\sigma_{33})^2+(\sigma_{33}-\sigma_{11})^2+6[\,(\sigma_{12})^2+(\sigma_{23})^2+(\sigma_{31})^2\,]}{2}} \tag{3}$$

where it is understood that the stress components represent the average stress in the matrix constituent material. Similar to the effective hardened yield strength, the effective stress is also a function of the effective plastic strain $\varepsilon_{p,eff}$ in that the instantaneous tangent modulus E_{tan} of the matrix constituent material is required to compute the stress components, and E_{tan} is dependent on effective plastic strain $\varepsilon_{p,eff}$ as shown in the equation below.

$$E_{tan} = \frac{EH}{E+H} \tag{4}$$

In Equation 4, H is the plastic modulus of the matrix constituent material and is expressed in terms of $\varepsilon_{p,eff}$ as follows.

$$H = \frac{E^{1/n}\ \sigma_o^{(n-1)/n}\ \varepsilon_{p,eff}{}^{(1-n)/n}}{n} \tag{5}$$

Thus, the determination of the plastic evolution that occurs during an imposed total strain increment reduces to finding the value of $\varepsilon_{p,eff}$ that allows the material state to remain on the expanding yield surface; i.e. we iteratively solve Eq. 2 for $\varepsilon_{p,eff}$.

The Ramberg-Osgood model (Eqs. 1-5) predicts a plasticity response that is isotropic; however, for short fiber filled, injection molded plastic parts that have a high degree of fiber alignment, the degree of plasticity exhibited prior to final rupture will depend strongly upon the direction of the loading relative to the average direction of the reinforcing fibers. The Ramberg-Osgood model can easily be enhanced to accommodate directional dependency by simply modifying the form of the effective stress (Eq. 3). In the present model, the modified effective stress is expressed as

$$\sigma_{eff} \equiv \sqrt{\frac{(\alpha\sigma_{11}-\beta\sigma_{22})^2+(\beta\sigma_{22}-\beta\sigma_{33})^2+(\beta\sigma_{33}-\alpha\sigma_{11})^2+6[\,(\sigma_{12})^2+(\sigma_{23})^2+(\sigma_{31})^2\,]}{2}} \tag{6}$$

where α and β are weighting coefficients that are used to differentiate the impact of stress components in the average fiber direction compared to stress components that are normal to the average fiber direction. In this directionally dependent formulation, the plasticity parameters that define the material response are σ_0, n, α and β.

It should be emphasized that the material only exhibits strong directional dependency when the degree of fiber alignedness is relatively high; when the fibers are randomly oriented, the plasticity response must revert to an isotropic response. To accommodate these behavioral characteristics, the directionally dependent weighting coefficients (α, β) cannot be constants, rather, they must be functions of the degree of fiber alignedness. In the present model, α and β are assumed to be linear functions of the degree of fiber alignedness which is quantified by the largest eigenvalue λ_I of the fiber orientation tensor.

$$\alpha(\lambda_I) = \theta + \left(\frac{(\alpha_m-\theta)}{(\lambda_{m,I}-1/2)}\right)(\lambda_I - 1/2) \tag{7}$$

$$\beta(\lambda_I) = \theta + \left(\frac{(\beta_m-\theta)}{(\lambda_{m,I}-1/2)}\right)(\lambda_I - 1/2) \tag{8}$$

In Equations 7 and 8, α_m and β_m are the values of α and β respectively that are optimized to fit the response characteristics of a strongly aligned material that has a largest fiber orientation eigenvalue of $\lambda_{m,I}$, and θ is the value that both α and β should achieve when the fiber orientation becomes random. Note that λ_I ranges from 1.0 for a perfectly aligned material to 1/3 for a material with completely random fiber orientation in all three directions (a completely isotropic condition). However, the condition of a truly 3-D random distribution of fibers is not likely to occur for thin injection molded parts. It is much more likely that a thin injection molded part might contain some locations that exhibit random fiber orientation in the plane of the injection molded part but not in the part's thickness direction (i.e., $\lambda_I = 1/2$). In order to accommodate this more common form of isotropy, Equations 7 and 8 contain the coefficient ($\lambda_I - 1/2$) which ensures that when $\lambda_I = 1/2$, α and β each achieve the same value of θ, and thus the material becomes isotropic.

The plasticity response of the matrix constituent material is defined by collectively by Eqs 1-8; however, the model must also utilize a rupture criterion that identifies complete failure of the short fiber filled material. It is assumed that the functional form of the effective stress expression (a weighted von Mises stress, see Eq. 6) is sufficient to define the directional dependency of the material for both the prediction of matrix plastic evolution and the prediction of matrix rupture. Therefore, the determination of the matrix rupture criterion requires that we simply establish an upper limit on the value of the weighted effective stress measure (denoted the effective strength S_{eff}). In this case, the matrix rupture condition is expressed as

$$S_{eff} \leq \sqrt{\frac{(\alpha\sigma_{11} - \beta\sigma_{22})^2 + (\beta\sigma_{22} - \beta\sigma_{33})^2 + (\beta\sigma_{33} - \alpha\sigma_{11})^2 + 6[\,(\sigma_{12})^2 + (\sigma_{23})^2 + (\sigma_{31})^2\,]}{2}} \tag{9}$$

where it is understood that the stress components represent the average stress in the matrix constituent material.

Once the matrix rupture condition is satisfied, several changes are made to the constitutive relations of the failed material. First, there is no longer any need to decompose the composite stress and strain into matrix stress and strain since we no longer have to compute plastic evolution for the matrix. Second, the constitutive relations of the homogenized composite material are used directly instead of building the composite constitutive relations from the constituent constitutive relations and microstructure (as seen earlier in Figure 3). Third, the stiffness of the composite material is instantaneously reduced to a user-specified fraction of the original elastic stiffness of the composite material. This stiffness reduction is performed by multiplying all six of the original composite moduli by the user-specified degradation constant. Note that once the rupture condition is triggered, the stiffness of the composite material remains fixed at the reduced value for the duration of the simulation. Finally, the composite constitutive relations are switched from a tangent formulation (which was used during the plasticity phase of the material response) to a secant formulation. Since the stiffness of the failed composite material remains fixed, the integration point in question no longer makes any direct contribution to the nonlinearity of the solution.

4. RESULTS

In order to use the multiscale material model that was described in Section 3 to model a specific short fiber filled material, we must first determine the value of the model's coefficients by fitting the model to a collection of experimental data for the material in question. Ideally, to allow for a definitive fitting of the model's coefficients, the collection of experimental data should cover the full range of behavior that can be exhibited by the material. However, from a practical point of view, it is highly desirable to limit both the number of different test types that have to be conducted and the complexity of the tests that have to be conducted. For the present model, good fits can be obtained by using three different tensile tests to failure. Figure 5 shows such a collection of tensile tests and includes data from tensile loading at three different directions relative to the average fiber direction of a mostly-aligned material. The tensile tests include 0° specimens, 45° specimens and 90° specimens. In each case, the specimen is loaded until final rupture occurs.

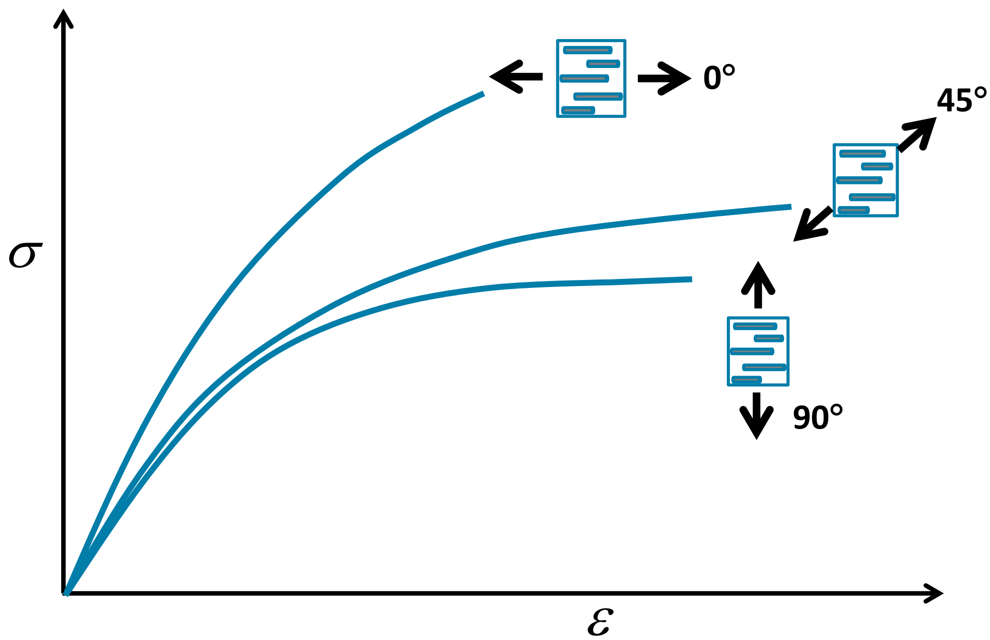

Figure 5. Typical collection of tensile tests to failure that are used to fit the coefficients of the present multiscale material model.

The model coefficients that must be determined include four elastic coefficients and four plastic coefficients. The elastic coefficients include the modulus and Poisson ratio of the matrix constituent and the modulus and Poisson ratio of the fiber constituent. The four plastic coefficients include σ_0, n, α and β. The model fitting process is performed in two steps. First, the four elastic coefficients are determined by requiring the model to accurately represent the initial elastic responses of the 0°, 90°, and 45° tensile test specimens (as provided by the first few data points from each of the three tensile tests. After the elastic coefficients have been determined, the second step involves fitting the four plastic coefficients to allow the model to accurately represent the full response history of all three tensile tests.

The process of establishing the effective strength that is needed for the rupture criterion (Eq. 9) begins with an examination of the 0°, 90°, and 45° tensile test data sets to identify the load level at which each specimen ruptures. These load levels will be denoted L_0, L_{90} and L_{45} for the 0°, 90° and 45° tensile test specimens respectively. Next, the model is used (with optimized elasticity and plasticity coefficients) to simulate each of the three tensile tests. For the 0° tensile test, when the model reaches the load level L_0, the model is used to compute the effective stress in the matrix that corresponds to the rupture event (this effective stress will the denoted the effective strength S_{eff_0}). Similarly, the model is used to simulate the 45° and 90° tensile tests to establish S_{eff_45} and S_{eff_90} respectively. If the weighted effective matrix stress was strongly relevant to the prediction of matrix rupture, we would expect that the values of S_{eff_0}, S_{eff_45} and S_{eff_90} would be similar. In fact, in all cases examined thus far, the range of the three values (S_{eff_0}, S_{eff_45}, S_{eff_90}) has been less than 10% of the mean value, thus indicating that the weighted effective matrix stress is indeed a good predictor of matrix rupture.

There are many ways that the effective strength S_{eff} (of Eq. 9) could be determined from the set of values S_{eff_0}, S_{eff_45} and S_{eff_90}. Here, we simply use $S_{eff} \equiv S_{eff\_0}$ because it is deemed important to ensure that the rupture condition of the 0° tensile test is accurately captured. It is anticipated that the injection molding process is designed to ensure that the primary tension/compression load paths have fibers that are mostly aligned with the load path. Therefore, the rupture of an injection molded part will be most accurately predicted if the 0° tensile test rupture condition can be accurately simulated.

Note that the rupture strength (Eq. 9) is formulated for a mostly-aligned material; in fact, a Moldflow simulation of the injection molded plaques that the tensile test specimens are cut from typically reveals that the fiber filled material exhibit a degree of alignedness of approximately $\lambda_I = 0.85$. However, the rupture criterion (Eq. 9) must account for the strength of the material regardless of the degree of fiber alignedness. Note that the rupture criterion (Eq. 9) depends on the directional plasticity parameters α and β which are in turn functions of λ_I . Thus the level of matrix stress that is required to trigger the rupture criterion changes with the degree of fiber alignedness (as quantified by the largest eigenvalue of the fiber orientation tensor).

Figures 6 through 8 show the results of fitting the multiscale material model to sets of 0°, 90°, and 45° tensile test data for three different, short fiber filled, injection molded plastics that collectively exhibit a wide range of stiffnesses, plastic response characteristics, and strengths. For each of the three materials, the model is able to closely match the plastic response and the rupture load of for all three load orientations (0°, 90°, and 45°).

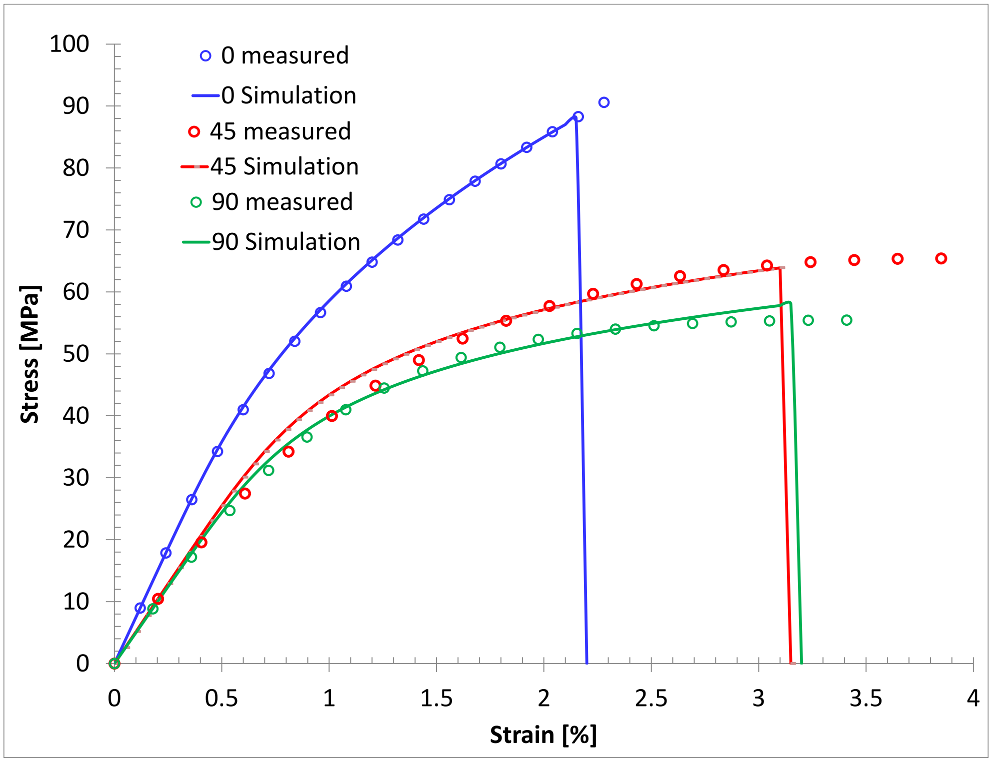

Figure 6. Material 1 – comparison of measured and predicted responses for tensile tests to failure at three different load orientations relative to the average fiber direction (0°, 90°, and 45°).

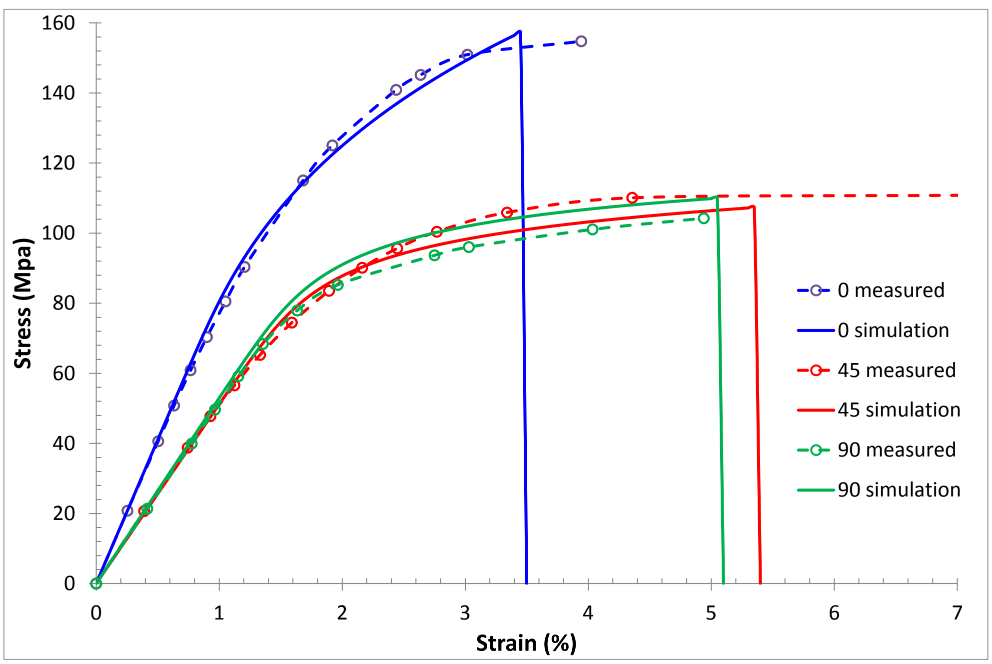

Figure 7. Material 2 – comparison of measured and predicted responses for tensile tests to failure at three different load orientations relative to the average fiber direction (0°, 90°, and 45°).

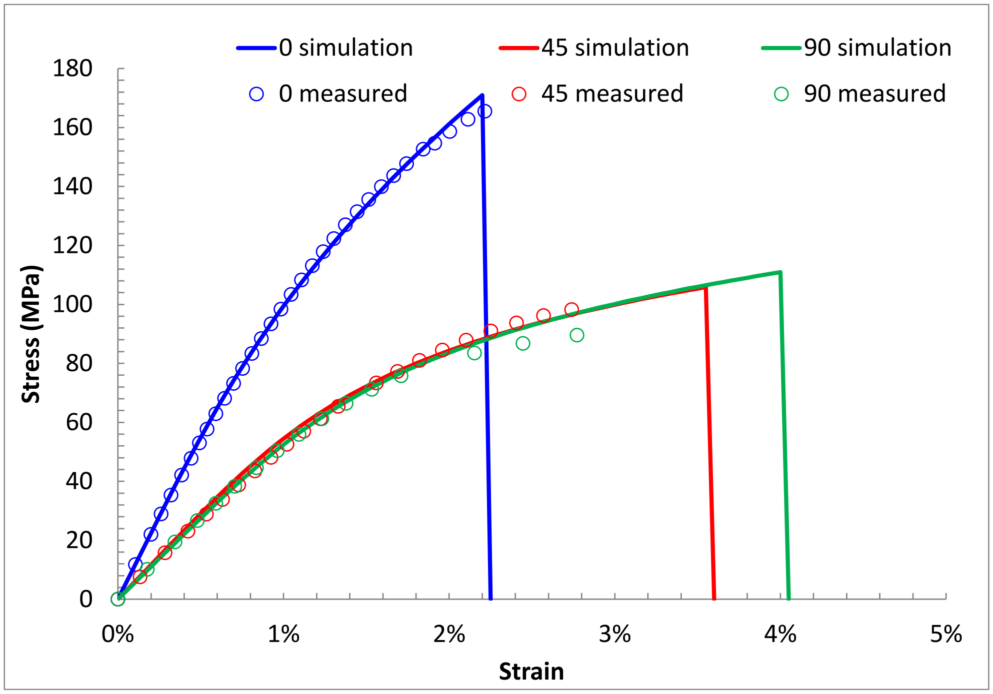

Figure 8. Material 3 – comparison of measured and predicted responses for tensile tests to failure at three different load orientations relative to the average fiber direction (0°, 90°, and 45°).

5. CONCLUSIONS

The use of short fiber reinforcing fillers has become common place in an effort to achieve higher stiffness-to-weight and higher strength-to-weight ratios for injection molded plastic parts. Autodesk is currently developing software that provides a seamless transition from the injection molding simulation to the nonlinear structural response simulation. Specifically, the software provides a seamless link between Autodesk Simulation Moldflow Insight (ASMI) and Autodesk Simulation Composites Analysis (ASCA). The key features of this software include:

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3. A robust material characterization process that uses relatively simple, measured experimental data of the short fiber filled plastic material to fit the parameters of the multiscale, progressive failure, constitutive model.

The multiscale, progressive failure, material model has proven to be capable of accurately representing the anisotropic plastic and rupture responses of numerous fiber filled materials under a wide range of load types. Such realistic material models will facilitate improved part optimization through increased accuracy of the FEA simulation models. It should be emphasized that in the current multiscale material model, it is assumed that physical debonding of the fiber/matrix interface can be adequately accounted for via plasticity of the matrix constituent material; however, it is possible to extend the incremental Mori-Tanaka model to explicitly account for fiber/matrix debonding as demonstrated by Jin.[14]

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